

Tau Physics: Theoretical Perspective

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The leptonic decays of the τ lepton provide relevant tests on the structure of the weak currents and the universality of their couplings to the gauge bosons. The hadronic τ decay modes constitute an ideal tool for studying low-energy effects of the strong interaction in very clean conditions. Accurate determinations of the QCD coupling and the strange quark mass have been obtained with τ decay data. New physics phenomena, such as a non-zero m_{ν_τ} or violations of conservation laws can also be searched for with τ decays.

1. INTRODUCTION

The τ lepton is a member of the third generation which decays into particles belonging to the first and second ones. Thus, τ physics could provide some clues to the puzzle of the recurring families of leptons and quarks. One naïvely expects the heavier fermions to be more sensitive to whatever dynamics is responsible for the fermion-mass generation. The leptonic or semileptonic character of τ decays provides a clean laboratory to test the structure of the weak currents and the universality of their couplings to the gauge bosons. Moreover, the τ is the only known lepton massive enough to decay into hadrons; its semileptonic decays are an ideal tool for studying strong interaction effects in very clean conditions.

The last few years have witnessed [1–3] a substantial change on our knowledge of the τ properties. The large (and clean) data samples collected by the most recent experiments have improved considerably the statistical accuracy and, moreover, have brought a new level of systematic understanding. On the theoretical side, the detailed study of higher-order electroweak corrections and QCD contributions has promoted the physics of the τ lepton to the level of precision tests.

2. UNIVERSALITY

2.1. Charged Currents

The leptonic decays $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau, \mu^- \bar{\nu}_\mu \nu_\tau$ are theoretically understood at the level of the electroweak radiative corrections [4]. Within the

Table 1

Average values [6–8] of some basic τ parameters.

m_τ	$(1777.03^{+0.30}_{-0.26}) \text{ MeV}$
τ_τ	$(290.89 \pm 1.00) \text{ fs}$
B_e	$(17.804 \pm 0.051)\%$
B_μ	$(17.336 \pm 0.051)\%$
$\text{Br}(\tau^- \rightarrow \nu_\tau \pi^-)$	$(11.03 \pm 0.14)\%$
$\text{Br}(\tau^- \rightarrow \nu_\tau K^-)$	$(0.684 \pm 0.022)\%$

Standard Model (SM),

$$\Gamma(\tau^- \rightarrow \nu_\tau l^- \bar{\nu}_l) = \frac{G_F^2 m_\tau^5}{192\pi^3} f(m_l^2/m_\tau^2) r_{EW}, \quad (1)$$

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$. The factor $r_{EW} = 0.9960$ takes into account radiative corrections not included in the Fermi coupling constant G_F , and the non-local structure of the W propagator [4].

Using the value of G_F measured in μ decay, $G_F = (1.16637 \pm 0.00001) \times 10^{-5} \text{ GeV}^{-2}$ [4,5], Eq. (1) provides a relation [2] between the leptonic branching ratios $B_l \equiv B(\tau^- \rightarrow \nu_\tau l^- \bar{\nu}_l)$ and the τ lifetime:

$$\begin{aligned} B_e &= \frac{B_\mu}{0.972564 \pm 0.000010} \\ &= \frac{\tau_\tau}{(1.6321 \pm 0.0014) \times 10^{-12} \text{ s}}. \end{aligned} \quad (2)$$

The errors reflect the present uncertainty of 0.3 MeV in the value of m_τ .

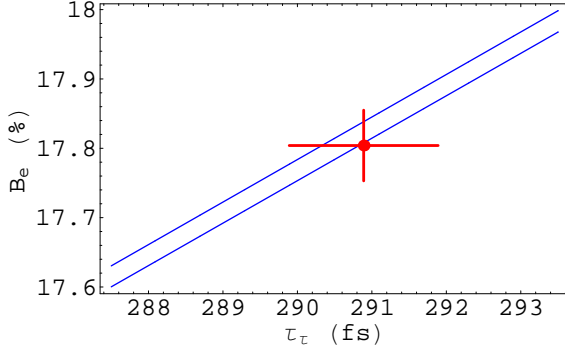


Figure 1. Relation between B_e and τ_τ . The band corresponds to the prediction in Eq. (2).

The relevant experimental measurements are given in Table 1. The predicted B_μ/B_e ratio is in perfect agreement with the measured value $B_\mu/B_e = 0.974 \pm 0.004$. As shown in Fig. 1, the relation between B_e and τ_τ is also well satisfied by the present data. The experimental precision (0.3%) is already approaching the level where a possible non-zero ν_τ mass could become relevant; the present bound [9] $m_{\nu_\tau} < 18.2$ MeV (95% CL) only guarantees that such effect is below 0.08%.

These measurements can be used to test the universality of the W couplings to the leptonic charged currents. The B_μ/B_e ratio constraints $|g_\mu/g_e|$, while B_e/τ_τ and B_μ/τ_τ provide information on $|g_\tau/g_\mu|$ and $|g_\tau/g_e|$, respectively. The present results are shown in Table 2, together with the values obtained from the ratios $\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)/\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$ [10] and $\Gamma(\tau^- \rightarrow \nu_\tau P^-)/\Gamma(P^- \rightarrow \mu^- \bar{\nu}_\mu)$ [$P = \pi, K$]. Also shown are the direct constraints obtained from the $W^- \rightarrow l^- \bar{\nu}_l$ decay modes at LEP II [11]. The present data verify the universality of the leptonic charged-current couplings to the 0.15% (e/μ) and 0.23% (τ/μ , τ/e) level.

2.2. Neutral Currents

In the SM, all leptons with equal electric charge have identical couplings to the Z boson. This has been tested at LEP and SLC [11], by measuring the total $e^+e^- \rightarrow Z \rightarrow l^+l^-$ cross-section, the forward-backward asymmetry, the (final) polar-

Table 2

Present constraints on $|g_l/g_{l'}|$.

	$ g_\mu/g_e $
B_μ/B_e	1.0006 ± 0.0021
$B_{\pi \rightarrow e}/B_{\pi \rightarrow \mu}$	1.0017 ± 0.0015
$B_{W \rightarrow \mu}/B_{W \rightarrow e}$	0.999 ± 0.013
	$ g_\tau/g_\mu $
$B_e \tau_\mu/\tau_\tau$	0.9995 ± 0.0023
$\Gamma_{\tau \rightarrow \pi}/\Gamma_{\pi \rightarrow \mu}$	1.005 ± 0.007
$\Gamma_{\tau \rightarrow K}/\Gamma_{K \rightarrow \mu}$	0.977 ± 0.016
$B_{W \rightarrow \tau}/B_{W \rightarrow \mu}$	1.022 ± 0.014
	$ g_\tau/g_e $
$B_\mu \tau_\mu/\tau_\tau$	1.0001 ± 0.0023
$B_{W \rightarrow \tau}/B_{W \rightarrow e}$	1.021 ± 0.015

ization asymmetry, the forward-backward (final) polarization asymmetry, and (at SLC) the left-right asymmetry between the cross-sections for initial left- and right-handed electrons and the left-right forward-backward asymmetry. The Z partial decay width to the l^+l^- final state determines the sum $(v_l^2 + a_l^2)$, while the ratio v_l/a_l is derived from the asymmetries, which measure the average longitudinal polarization of the lepton l^- : $\mathcal{P}_l \equiv -2v_l a_l / (v_l^2 + a_l^2)$.

The measurement of the final polarization asymmetries can (only) be done for $l = \tau$, because the spin polarization of the τ 's is reflected in the distorted distribution of their decay products. Therefore, \mathcal{P}_τ and \mathcal{P}_e can be determined from a measurement of the spectrum of the final charged particles in the decay of one τ , or by studying the correlated distributions between the final products of both τ 's [12,13].

The data are in excellent agreement with the SM predictions and confirm the universality of the leptonic neutral couplings. Figure 2 shows the 68% probability contours in the a_l-v_l plane, obtained from a combined analysis [11] of all leptonic observables.

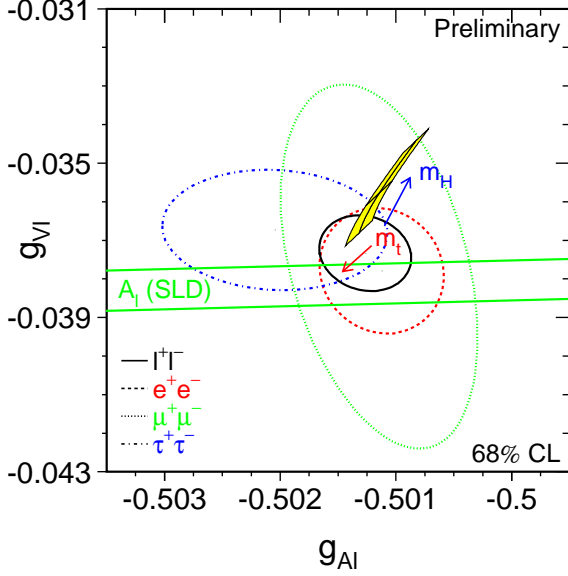


Figure 2. 68% probability contours in the a_l - ν_l plane from LEP data [11]. Also shown is the 1σ band resulting from the A_{LR}^0 measurement at SLD. The grid corresponds to the SM prediction.

3. LORENTZ STRUCTURE

Let us consider the leptonic decay $l^- \rightarrow \nu_l l'^- \bar{\nu}_{l'}$. The most general, local, derivative-free, lepton-number conserving, four-lepton interaction Hamiltonian, consistent with locality and Lorentz invariance [14,15],

$$\mathcal{H} = 4 \frac{G_{l'l}}{\sqrt{2}} \sum_{n, \epsilon, \omega} g_{\epsilon\omega}^n [\bar{l}' \Gamma^n (\nu_{l'})_\sigma] \left[\overline{(\nu_l)_\lambda} \Gamma_n l_\omega \right], \quad (3)$$

contains ten complex coupling constants or, since a common phase is arbitrary, nineteen independent real parameters which could be different for each leptonic decay. The subindices $\epsilon, \omega, \sigma, \lambda$ label the chiralities (left-handed, right-handed) of the corresponding fermions, and n the type of interaction: scalar (I), vector (γ^μ), tensor ($\sigma^{\mu\nu}/\sqrt{2}$). For given n, ϵ, ω , the neutrino chiralities σ and λ are uniquely determined.

The total decay width is proportional to the following combination of couplings, which is usu-

ally normalized to one [15]:

$$\begin{aligned} 1 &= \frac{1}{4} (|g_{RR}^S|^2 + |g_{RL}^S|^2 + |g_{LR}^S|^2 + |g_{LL}^S|^2) \\ &\quad + (|g_{RR}^V|^2 + |g_{RL}^V|^2 + |g_{LR}^V|^2 + |g_{LL}^V|^2) \\ &\quad + 3 (|g_{RL}^T|^2 + |g_{LR}^T|^2) \\ &\equiv Q_{RR} + Q_{RL} + Q_{LR} + Q_{LL}. \end{aligned} \quad (4)$$

The universality tests mentioned before refer then to the global normalization $G_{l'l}$, while the $g_{\epsilon\omega}^n$ couplings parameterize the relative strength of different types of interaction. The sums $Q_{\epsilon\omega}$ of all factors with the same subindices give the probability of having a decay from an initial charged lepton with chirality ω to a final one with chirality ϵ . In the SM, $g_{LL}^V = 1$ and all other $g_{\epsilon\omega}^n = 0$.

The energy spectrum and angular distribution of the final charged lepton provides information on the couplings $g_{\epsilon\omega}^n$. For μ decay, where precise measurements of the polarizations of both μ and e have been performed, there exist [15] upper bounds on Q_{RR} , Q_{LR} and Q_{RL} , and a lower bound on Q_{LL} . They imply corresponding upper bounds on the 8 couplings $|g_{RR}^n|$, $|g_{LR}^n|$ and $|g_{RL}^n|$. The measurements of the μ^- and the e^- do not allow to determine $|g_{LL}^S|$ and $|g_{LL}^V|$ separately [15,16]. Nevertheless, since the helicity of the ν_μ in pion decay is experimentally known [17] to be -1 , a lower limit on $|g_{LL}^V|$ is obtained [15] from the inverse muon decay $\nu_\mu e^- \rightarrow \mu^- \nu_e$. These limits show nicely that the bulk of the μ -decay transition amplitude is indeed of the predicted V-A type: $|g_{LL}^V| > 0.960$ (90% CL) [7]. Improved bounds on the μ couplings are expected from the Twist experiment [18] at TRIUMF.

The experimental analysis of the τ -decay parameters is necessarily different from the one applied to the muon, because of the much shorter τ lifetime. The measurement of the τ polarization is still possible due to the fact that the spins of the $\tau^+ \tau^-$ pair produced in $e^+ e^-$ annihilation are strongly correlated [12,13,19]. Another possibility is to use the beam polarization, as done by SLD. However, the polarization of the charged lepton emitted in the τ decay has never been measured. The experimental study of the inverse decay $\nu_\tau l^- \rightarrow \tau^- \nu_l$ looks far out of reach.

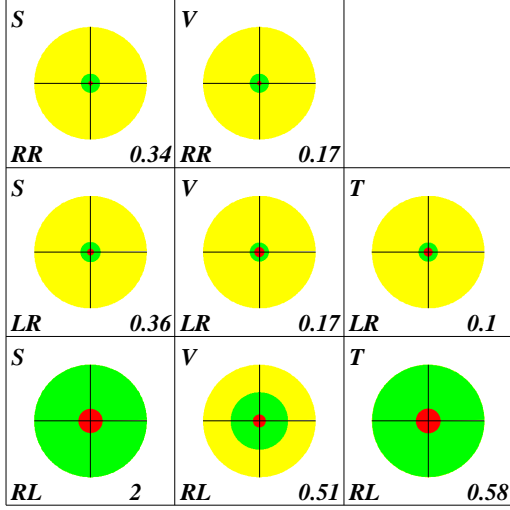


Figure 3. 90% CL limits [20] for the normalized τ -decay couplings $g_{\epsilon\omega}^n \equiv g_{\epsilon\omega}^n/N^n$, where $N^n \equiv \max(|g_{\epsilon\omega}^n|) = 2, 1, 1/\sqrt{3}$ for $n = S, V, T$, assuming e/μ universality. The circles of unit area indicate the range allowed by the normalization constraint (4). The experimental bounds are shown as shaded circles. The μ -decay limits are also shown (darker circles).

The determination of the τ polarization parameters allows us to bound the total probability for the decay of a right-handed τ , $Q_{\tau_R} \equiv Q_{RR} + Q_{LR}$. At 90% CL, one finds: $Q_{\tau_R}^{\tau \rightarrow \mu} < 0.047$, $Q_{\tau_R}^{\tau \rightarrow e} < 0.054$ and $Q_{\tau_R}^{\tau \rightarrow l} < 0.032$, where the last value refers to the τ decay into either $l = e$ or μ , assuming identical e/μ couplings. These positive semidefinite probabilities imply corresponding limits on all $|g_{RR}^n|$ and $|g_{LR}^n|$ couplings. Including also the information from the energy distribution, one gets the bounds shown in Fig. 3, where e/μ universality has been assumed.

4. SEARCHING FOR NEW PHYSICS

4.1. The Tau Neutrino

The DONUT experiment at Fermilab has provided [21] the first direct observation of the ν_τ (produced through $p + N \rightarrow D_s + \dots$, followed

by the decays $D_s \rightarrow \tau^- \bar{\nu}_\tau$ and $\tau^- \rightarrow \nu_\tau + \dots$), through the detection of $\nu_\tau + N \rightarrow \tau + X$. With this important achievement, all SM fermions have been finally detected and the three-family structure is definitely established.

The feasibility to detect τ neutrinos is of great importance, in view of the recent SuperKamiokande results [22] suggesting $\nu_\mu \rightarrow \nu_\tau$ oscillations with $m_{\nu_\tau}^2 - m_{\nu_\mu}^2 \sim (0.05 \text{ eV})^2$. This hypothesis could be corroborated making a long-baseline neutrino experiment with a ν_μ beam pointing into a far ($\sim 700 \text{ Km}$) massive detector, able to detect the appearance of a τ [23].

All observed τ decays are supposed to be accompanied by neutrino emission, in order to fulfil energy-momentum conservation requirements. From a two-dimensional likelihood fit of the visible energy and the invariant-mass distribution of the final hadrons in $\tau^- \rightarrow \nu_\tau X^-$ events, it is possible to set a limit on the ν_τ mass [24]. The strongest bound up to date [9],

$$m_{\nu_\tau} < 18.2 \text{ MeV} \quad (95\% \text{ CL}), \quad (5)$$

has been obtained from a combined analysis of $\tau^- \rightarrow (3\pi)^- \nu_\tau, (5\pi)^- \nu_\tau$ events.

4.2. Lepton-Number Violation

In the minimal SM with massless neutrinos, there is a separately conserved additive lepton number for each generation. All present data are consistent with this conservation law. However, there are no strong theoretical reasons forbidding a mixing among the different leptons, in the same way as happens in the quark sector. Many models in fact predict lepton-flavour or even lepton-number violation at some level [25]. Experimental searches for these processes can provide information on the scale at which the new physics begins to play a significant role.

The present upper limits on lepton-flavour and lepton-number violating decays of the τ [26] are in the range of 10^{-5} to 10^{-6} , which is far away from the impressive bounds [7] obtained in μ decay [$\text{Br}(\mu^- \rightarrow e^- \gamma) < 1.2 \times 10^{-11}$, $\text{Br}(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \times 10^{-12}$, $\text{Br}(\mu^- \rightarrow e^- \gamma \gamma) < 7.2 \times 10^{-11}$ (90% CL)]. Nevertheless, the τ -decay limits start to put interesting constraints on possible new physics contributions. With future τ

samples of 10^7 events per year, an improvement of two orders of magnitude would be possible.

4.3. Dipole Moments

Owing to their chiral-changing structure, the electroweak dipole moments may provide important insights on the mechanism responsible for mass generation. In general, one expects that a fermion of mass m_f (generated by physics at some scale $M \gg m_f$) will have induced dipole moments proportional to some power of m_f/M . Therefore, heavy fermions such as the τ should be a good testing ground for this kind of effects. Of special interest [27] are the electric and weak dipole moments, $d_\tau^{\gamma,Z}$, which violate T and P invariance; they constitute a good probe of CP violation.

The present experimental constraints on the electroweak dipole moments of the τ have been recently reanalyzed, using effective operator techniques [28]. The achieved sensitivity is still marginal, but it is approaching the level of the SM contribution to the τ anomalous magnetic moment [29]: $a_\tau^\gamma|_{\text{th}} = (1.1773 \pm 0.0003) \times 10^{-3}$.

5. HADRONIC DECAYS

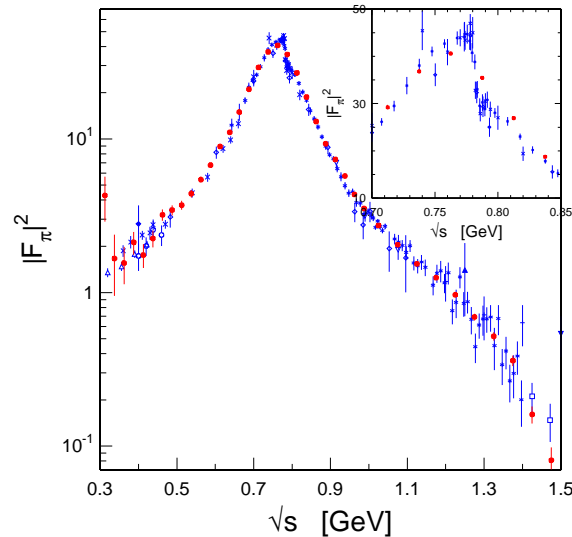


Figure 4. Pion form factor from $\tau^- \rightarrow \nu_\tau \pi^- \pi^0$ [30] (filled circles) and $e^+e^- \rightarrow \pi^+\pi^-$ data.

The semileptonic decay modes $\tau^- \rightarrow \nu_\tau H^-$ probe the matrix element of the left-handed charged current between the vacuum and the final hadronic state H^- .

For the decay modes with lowest multiplicity, $\tau^- \rightarrow \nu_\tau \pi^-$ and $\tau^- \rightarrow \nu_\tau K^-$, the relevant matrix elements are already known from the measured decays $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ and $K^- \rightarrow \mu^- \bar{\nu}_\mu$. The corresponding τ decay widths can then be predicted rather accurately. As shown in Table 2, these predictions are in good agreement with the measured values, and provide a quite precise test of charged-current universality.

For the two-pion final state, the hadronic matrix element is parameterized in terms of the so-called pion form factor:

$$\langle \pi^- \pi^0 | \bar{d} \gamma^\mu u | 0 \rangle \equiv \sqrt{2} F_\pi(s) (p_{\pi^-} - p_{\pi^0})^\mu. \quad (6)$$

Figure 4 shows the recent CLEO measurement of $|F_\pi(s)|^2$ from $\tau \rightarrow \nu_\tau \pi^- \pi^0$ data [30] (a similar analysis was done previously by ALEPH [31]). Also shown is the corresponding determination from $e^+e^- \rightarrow \pi^+\pi^-$ data. The precision achieved with τ decays is clearly better. There is good agreement between both sets of data, although the τ points tend to be slightly higher [32].

A dynamical understanding of the pion form factor can be achieved [33–35], by using analyticity, unitarity and some general properties of QCD.

At low momentum transfer, the coupling of any number of π 's, K 's and η 's to the $V-A$ current can be rigorously calculated with Chiral Perturbation Theory [36,37] techniques, as an expansion in powers of s and light quark masses over the chiral symmetry breaking scale ($\Lambda_\chi \sim 1$ GeV). This includes chiral loop corrections, which encode the absorptive contributions required by unitarity. The short-distance information is contained in the so-called chiral couplings, which are known to be dominated by the effect of the lowest-mass resonances [38].

In the limit of an infinite number of quark colours N_C , QCD reduces to a theory of tree-level resonance exchanges [39]. Thus, the ρ propagator governs the pion form factor at $\sqrt{s} \lesssim 1$ GeV, providing an all-order resummation of the polynomial chiral corrections. Moreover, requir-

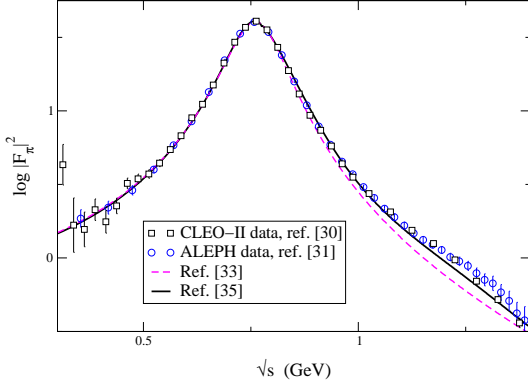


Figure 5. Pion form factor data compared with theoretical predictions [35].

ing $F_\pi(s)$ to satisfy the correct QCD behaviour at large s , one can determine the relevant ρ couplings [38]. The leading $1/N_C$ corrections correspond to pion loops and can be incorporated by matching the large- N_C result with the Chiral Perturbation Theory description [33]. Using analyticity and unitarity constraints, the chiral logarithms associated with those pion loops can be exponentiated to all orders in the chiral expansion. Putting all these fundamental ingredients together, one gets the result [33]:

$$F_\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp \left\{ -\frac{s \operatorname{Re} A(s)}{96\pi^2 f_\pi^2} \right\},$$

where

$$A(s) \equiv \log \left(\frac{m_\pi^2}{M_\rho^2} \right) + 8 \frac{m_\pi^2}{s} - \frac{5}{3} + \sigma_\pi^3 \log \left(\frac{\sigma_\pi + 1}{\sigma_\pi - 1} \right)$$

contains the one-loop chiral logarithms, $\sigma_\pi \equiv \sqrt{1 - 4m_\pi^2/s}$ and the off-shell ρ width is given by [33,34] $\Gamma_\rho(s) = \theta(s - 4m_\pi^2) \sigma_\pi^3 M_\rho s / (96\pi f_\pi^2)$. This prediction, which only depends on M_ρ , m_π and the pion decay constant f_π , is compared with the data in Fig. 5. The agreement is rather impressive and extends to negative s values, where the $e^- \pi$ elastic data (not shown in the figure) sits.

One can include the effect of higher ρ resonances, at the price of having some free parameters (subtraction constants) which decrease the

predictive power [35]. This gives a better description of the ρ' shoulder around 1.2 GeV.

The dynamical structure of other hadronic final states has been also investigated. CLEO has measured [40] the four $J^P = 1^+$ structure functions characterizing the decay $\tau^- \rightarrow \nu_\tau \pi^- 2\pi^0$, improving a previous OPAL analysis [41]. A theoretical analysis of these data is in progress [42].

6. THE TAU HADRONIC WIDTH

The inclusive character of the total τ hadronic width renders possible an accurate calculation of the ratio [43–47]

$$R_\tau \equiv \frac{\Gamma[\tau^- \rightarrow \nu_\tau \text{ hadrons}(\gamma)]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]}, \quad (7)$$

using analyticity constraints and the Operator Product Expansion (OPE).

The theoretical analysis of R_τ involves the two-point correlation functions

$$\Pi_j^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | T(j^\mu(x) j^\nu(0)^\dagger) | 0 \rangle \quad (8)$$

for the vector, $j^\mu = V_{ij}^\mu \equiv \bar{\psi}_j \gamma^\mu \psi_i$, and axial-vector, $j^\mu = A_{ij}^\mu \equiv \bar{\psi}_j \gamma^\mu \gamma_5 \psi_i$, colour-singlet quark currents ($i, j = u, d, s$). They have the Lorentz decompositions

$$\begin{aligned} \Pi_{ij,V/A}^{\mu\nu}(q) = & (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{ij,V/A}^{(1)}(q^2) \\ & + q^\mu q^\nu \Pi_{ij,V/A}^{(0)}(q^2), \end{aligned} \quad (9)$$

where the superscript $J = 0, 1$ denotes the angular momentum in the hadronic rest frame.

The imaginary parts of the two-point functions $\Pi_{ij,V/A}^{(J)}(q^2)$ are proportional to the spectral functions for hadrons with the corresponding quantum numbers. The hadronic decay rate of the τ can be written as an integral of these spectral functions over the invariant mass s of the final-state hadrons:

$$\begin{aligned} R_\tau = & 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2} \right)^2 \\ & \times \left[\left(1 + 2 \frac{s}{m_\tau^2} \right) \operatorname{Im} \Pi^{(1)}(s) + \operatorname{Im} \Pi^{(0)}(s) \right]. \end{aligned} \quad (10)$$

The appropriate combinations of correlators are

$$\begin{aligned} \Pi^{(J)}(s) \equiv & |V_{ud}|^2 \left(\Pi_{ud,V}^{(J)}(s) + \Pi_{ud,A}^{(J)}(s) \right) \\ & + |V_{us}|^2 \left(\Pi_{us,V}^{(J)}(s) + \Pi_{us,A}^{(J)}(s) \right). \end{aligned} \quad (11)$$

We can separate the inclusive contributions associated with specific quark currents:

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}. \quad (12)$$

$R_{\tau,V}$ and $R_{\tau,A}$ correspond to the first two terms in (11), while $R_{\tau,S}$ contains the remaining Cabibbo-suppressed contributions. Non-strange hadronic decays of the τ are resolved experimentally into vector ($R_{\tau,V}$) and axial-vector ($R_{\tau,A}$) contributions according to whether the hadronic final state includes an even or odd number of pions. Strange decays ($R_{\tau,S}$) are of course identified by the presence of an odd number of kaons in the final state.

Since the hadronic spectral functions are sensitive to the non-perturbative effects of QCD that bind quarks into hadrons, the integrand in Eq. (10) cannot be calculated at present from QCD. Nevertheless the integral itself can be calculated systematically by exploiting the analytic properties of the correlators $\Pi^{(J)}(s)$. They are analytic functions of s except along the positive real s -axis, where their imaginary parts have discontinuities. R_τ can therefore be expressed as a contour integral in the complex s -plane running counter-clockwise around the circle $|s| = m_\tau^2$:

$$\begin{aligned} R_\tau = & 6\pi i \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2} \right)^2 \\ & \times \left[\left(1 + 2\frac{s}{m_\tau^2} \right) \Pi^{(0+1)}(s) - 2\frac{s}{m_\tau^2} \Pi^{(0)}(s) \right]. \end{aligned} \quad (13)$$

This expression requires the correlators only for complex s of order m_τ^2 , which is significantly larger than the scale associated with non-perturbative effects in QCD. The short-distance OPE can therefore be used to organize the perturbative and non-perturbative contributions to the correlators into a systematic expansion [48] in powers of $1/s$. The possible uncertainties associated with the use of the OPE near the time-like axis are negligible in this case, because the integrand in (13) includes a factor $(1 - s/m_\tau^2)^2$, which

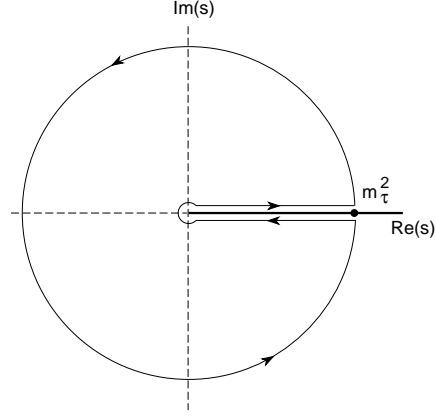


Figure 6. Integration contour in the complex s -plane, used to obtain Eq. (13)

provides a double zero at $s = m_\tau^2$, effectively suppressing the contribution from the region near the branch cut.

After evaluating the contour integral, R_τ can be expressed as an expansion in powers of $1/m_\tau^2$, with coefficients that depend only logarithmically on m_τ :

$$R_\tau = 3 S_{EW} \left\{ 1 + \delta'_{EW} + \sum_{D=0,2,\dots} \delta^{(D)} \right\}. \quad (14)$$

The factors $S_{EW} = 1.0194$ and $\delta'_{EW} = 0.0010$ contain the known electroweak corrections at the leading [4] and next-to-leading [49] logarithm approximation. The dimension-0 contribution, $\delta^{(0)}$, is the purely perturbative correction neglecting quark masses. It is given by [43–47]:

$$\begin{aligned} \delta^{(0)} &= \sum_{n=1} K_n A^{(n)}(\alpha_s) \\ &= a_\tau + 5.2023 a_\tau^2 + 26.366 a_\tau^3 + \mathcal{O}(a_\tau^4), \end{aligned} \quad (15)$$

where $a_\tau \equiv \alpha_s(m_\tau^2)/\pi$.

The dynamical coefficients K_n regulate the perturbative expansion of $-s \frac{d}{ds} \Pi^{(0+1)}(s)$ in the massless-quark limit [$s \Pi^{(0)}(s) = 0$ for massless quarks]; they are known [50,51] to $\mathcal{O}(\alpha_s^3)$: $K_1 = 1$; $K_2 = 1.6398$; $K_3(\overline{MS}) = 6.3710$. The kinematical effect of the contour integration is

contained in the functions [46]

$$A^{(n)}(\alpha_s) = \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \frac{ds}{s} \left(\frac{\alpha_s(-s)}{\pi} \right)^n \times \left(1 - 2\frac{s}{m_\tau^2} + 2\frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8} \right), \quad (16)$$

which only depend on $\alpha_s(m_\tau^2)$. Owing to the long running of the strong coupling along the circle, the coefficients of the perturbative expansion of $\delta^{(0)}$ in powers of $\alpha_s(m_\tau^2)$ are larger than the direct K_n contributions. This running effect can be properly resummed to all orders in α_s by fully keeping [46] the known four-loop-level calculation of the integrals $A^{(n)}(\alpha_s)$.

The leading quark-mass corrections $\delta^{(2)}$ are tiny for the up and down quarks. The correction from the strange quark mass is important for strange decays but, owing to the $|V_{us}|^2$ suppression, the effect on the total ratio R_τ is below 1%.

The non-perturbative contributions can be shown to be suppressed by six powers of the τ mass [45] and, therefore, are very small. Their numerical size has been determined from the invariant-mass distribution of the final hadrons in τ decay, through the study of weighted integrals [52],

$$R_\tau^{kl} \equiv \int_0^{m_\tau^2} ds \left(1 - \frac{s}{m_\tau^2} \right)^k \left(\frac{s}{m_\tau^2} \right)^l \frac{dR_\tau}{ds}, \quad (17)$$

which can be calculated theoretically in the same way as R_τ . The predicted suppression [45] of the non-perturbative corrections has been confirmed by ALEPH [53], CLEO [54] and OPAL [55]. The most recent analyses [53,55] give

$$\delta_{\text{NP}} \equiv \sum_{D \geq 4} \delta^{(D)} = -0.003 \pm 0.003. \quad (18)$$

The QCD prediction for $R_{\tau,V+A}$ is then completely dominated by the perturbative contribution $\delta^{(0)}$; non-perturbative effects being smaller than the perturbative uncertainties from uncalculated higher-order corrections. The result turns out to be very sensitive to the value of $\alpha_s(m_\tau^2)$, allowing for an accurate determination of the fundamental QCD coupling. The experimental measurement [53,55] $R_{\tau,V+A} = 3.484 \pm 0.024$ implies

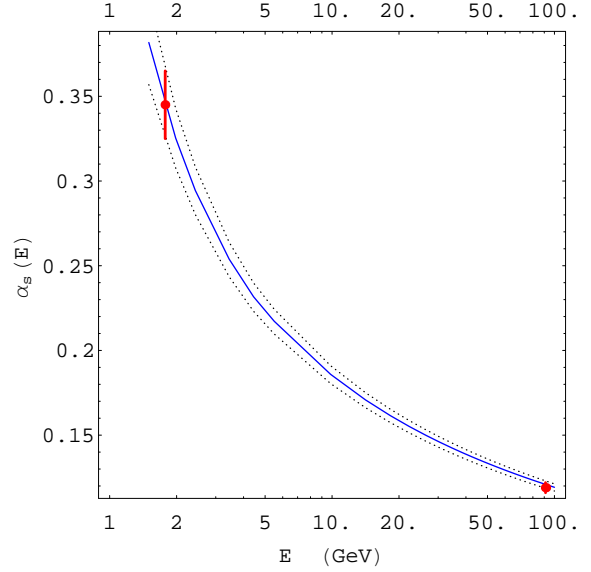


Figure 7. Measured values of α_s in τ and Z decays. The curves show the energy dependence predicted by QCD, using $\alpha_s(m_\tau^2)$ as input.

$\delta^{(0)} = 0.200 \pm 0.013$, which corresponds (in the $\overline{\text{MS}}$ scheme) to

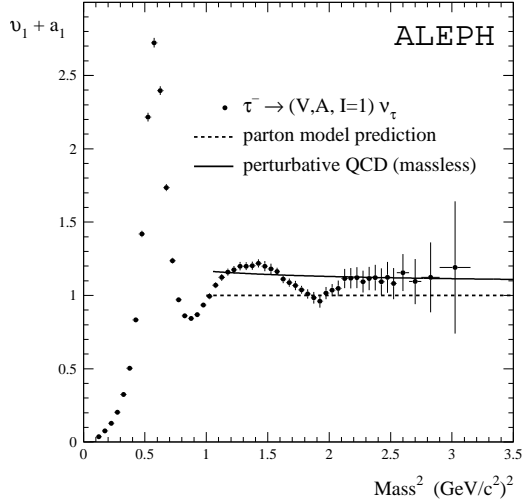
$$\alpha_s(m_\tau^2) = 0.345 \pm 0.020. \quad (19)$$

The strong coupling measured at the τ mass scale is significantly different from the values obtained at higher energies. From the hadronic decays of the Z boson, one gets $\alpha_s(M_Z^2) = 0.119 \pm 0.003$ [7,11], which differs from the τ decay measurement by eleven standard deviations. After evolution up to the scale M_Z [56], the strong coupling constant in (19) decreases to

$$\alpha_s(M_Z^2) = 0.1208 \pm 0.0025, \quad (20)$$

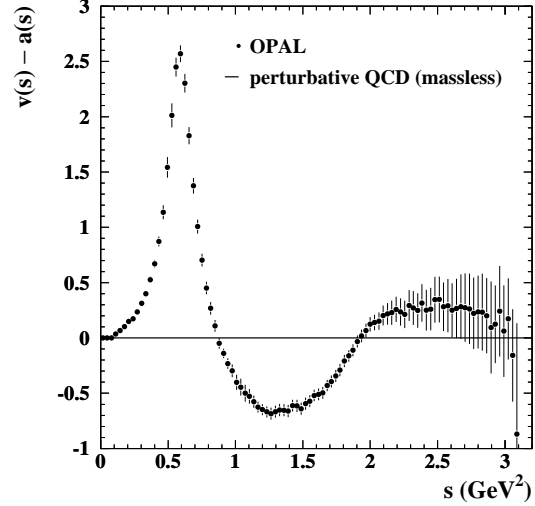
in excellent agreement with the direct measurements at the Z peak and with a similar accuracy. The comparison of these two determinations of α_s in two extreme energy regimes, m_τ and M_Z , provides a beautiful test of the predicted running of the QCD coupling; i.e. a very significant experimental verification of *asymptotic freedom*.

From a careful analysis of the hadronic invariant-mass distribution, ALEPH [31,53] and

Figure 8. $V + A$ spectral function [53].

OPAL [55] have measured the spectral functions associated with the vector and axial-vector quark currents. Their difference is a pure non-perturbative quantity, which carries important information on the QCD dynamics [45,57–60]; it allows to determine low-energy parameters, such as the pion decay constant, the electromagnetic pion mass difference $m_{\pi^\pm} - m_{\pi^0}$, or the axial pion form factor, in good agreement with their direct measurements [61].

The vector spectral function has been also used to measure the hadronic vacuum polarization effects associated with the photon and, therefore, estimate how the electromagnetic fine structure constant gets modified at LEP energies. The uncertainty of this parameter is one of the main limitations in the extraction of the Higgs mass from global electroweak fits to the LEP/SLD data. From the ALEPH τ data [31], the Orsay group obtains [61,62] $\alpha^{-1}(M_Z) = 128.933 \pm 0.021$, which reduces the error of the fitted $\log(M_H)$ value by 30%. The same τ data allows to pin down the hadronic contribution to the anomalous magnetic moment of the muon a_μ^γ . The recent analyses [30,62] have improved the theoretical prediction of a_μ^γ , setting a reference value to be compared

Figure 9. $V - A$ spectral function [55].

with the measurement of the BNL-E821 experiment, presently running at Brookhaven.

7. THE STRANGE QUARK MASS

The LEP experiments and CLEO have performed an extensive investigation of kaon production in τ decays [63,64]. ALEPH has determined the inclusive invariant mass distribution of the final hadrons in the Cabibbo-suppressed decays [64]. The separate measurement of the $|\Delta S| = 0$ and $|\Delta S| = 1$ decay widths allows us to pin down the SU(3) breaking effect induced by the strange quark mass, through the differences

$$\begin{aligned} \delta R_\tau^{kl} &\equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \\ &\approx 24 \frac{m_s^2(m_\tau^2)}{m_\tau^2} \Delta_{kl}(\alpha_s) - 48\pi^2 \frac{\delta O_4}{m_\tau^4} Q_{kl}(\alpha_s), \end{aligned} \quad (21)$$

where $\Delta_{kl}(\alpha_s)$ and $Q_{kl}(\alpha_s)$ are perturbative QCD corrections, which are known to $O(\alpha_s^3)$ and $O(\alpha_s^2)$, respectively [65]. The small non-perturbative contribution, $\delta O_4 \equiv \langle 0 | m_s \bar{s}s - m_d \bar{d}d | 0 \rangle = -(1.5 \pm 0.4) \times 10^{-3} \text{ GeV}^4$, has been estimated with Chiral Perturbation Theory techniques [65]. Table 3 shows the measured [64,66]

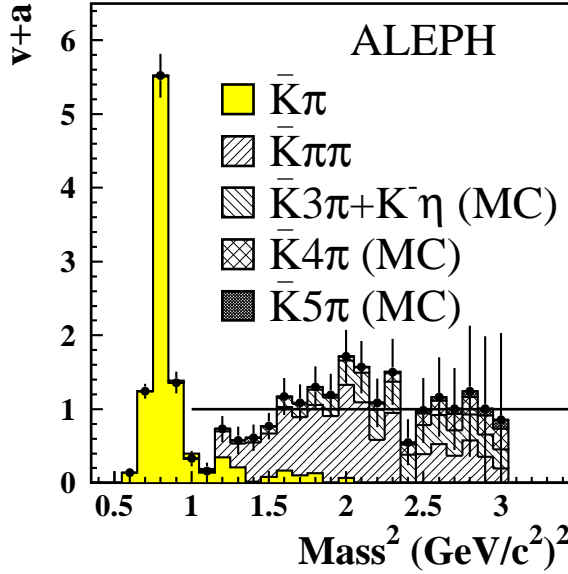


Figure 10. $|\Delta S| = 1$ spectral function [64].

Table 3

Measured moments δR_τ^{kl} [64,66] and corresponding $m_s(m_\tau^2)$ values [66].

(k, l)	δR_τ^{kl}	$m_s(m_\tau^2)$ (MeV)
(0, 0)	0.370 ± 0.133	$131 \pm 29_{\text{exp}} \pm 14_{\text{th}}$
(1, 0)	0.396 ± 0.078	$119 \pm 16_{\text{exp}} \pm 12_{\text{th}}$
(2, 0)	0.397 ± 0.054	$104 \pm 11_{\text{exp}} \pm 19_{\text{th}}$

differences δR_τ^{kl} and the corresponding $(\overline{\text{MS}})$ values [66] of $m_s(m_\tau^2)$. The theoretical errors are dominated by the very large perturbative uncertainties of $\Delta_{kl}(\alpha_s)$ [65,67–70].

A global analysis, using the information from the three moments and taking into account the strong error correlations, gives the result [66]

$$m_s(m_\tau^2) = (112 \pm 23) \text{ MeV}.$$

This corresponds to $m_s(1 \text{ GeV}^2) = (150 \pm 35) \text{ MeV}$. A similar result is obtained from an analysis based on “optimal moments”, with improved perturbative convergence [71].

Previous estimates of m_s were based on lattice

simulations or phenomenological QCD sum rules. There is a rather large spread of lattice results [72]; the average value agrees with the τ determination, but some results are too small and could be in conflict with QCD lower bounds [73]. The latest QCD sum rules [74] results are compatible with the τ value. The advantage of the τ determination is the direct use of experimental input, which makes easier to quantify the associated uncertainties.

8. SUMMARY

The flavour structure of the SM is one of the main pending questions in our understanding of weak interactions. Although we do not know the reason of the observed family replication, we have learned experimentally that the number of SM fermion generations is just three (and no more). Therefore, we must study as precisely as possible the few existing flavours to get some hints on the dynamics responsible for their observed structure.

The τ turns out to be an ideal laboratory to test the SM. It is a lepton, which means clean physics, and moreover it is heavy enough to produce a large variety of decay modes. Naïvely, one would expect the τ to be much more sensitive than the e or the μ to new physics related to the flavour and mass-generation problems.

QCD studies can also benefit a lot from the existence of this heavy lepton, able to decay into hadrons. Owing to their semileptonic character, the hadronic τ decays provide a powerful tool to investigate the low-energy effects of the strong interactions in rather simple conditions.

Our knowledge of the τ properties has been considerably improved during the last few years. Lepton universality has been tested to rather good accuracy, both in the charged and neutral current sectors. The Lorentz structure of the leptonic τ decays is certainly not determined, but begins to be experimentally explored. An upper limit of 3.2% (90% CL) has been already set on the probability of having a (wrong) decay from a right-handed τ . The quality of the hadronic data has made possible to perform quantitative QCD tests and determine the strong coupling constant very accurately. Searches for non-standard phe-

nomena have been pushed to the limits that the existing data samples allow to investigate.

At present, all experimental results on the τ lepton are consistent with the SM. There is, however, large room for improvements. Future τ experiments will probe the SM to a much deeper level of sensitivity and will explore the frontier of its possible extensions.

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